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Development of the method of generalized coordinates for solving non-linear non-stationary combined heat and mass transfer problems with complex boundary conditions

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Abstract—The method of generalized coordinates is developed in the application to the solution of non-linear non-stationary combined heat and mass transfer problems with complex boundary conditions. A good agreement is observed between the solutions under the first-kind boundary conditions and those obtained by other methods. The solution to the problem under the time-dependent third-kind boundary conditions and with the parabolic initial distribution potentials is given.

1. INTRODUCTION

CURRENT investigations into the phenomena of combined energy (heat) and mass transfer presume allowance for both the interrelation between these phenomena and the dependence of the physical characteristics of the medium on the magnitude of transfer potentials. It is a well-known fact, for example, that at high temperatures the thermal conductivity of metals and alloys is temperature dependent and that for some substances the diffusion and temperature-gradient coefficients depend on temperature and moisture content even at low temperatures. The dependence of the thermal conductivity coefficient of the medium on its moisture content during drying of materials is also well known. Analogous effects are also frequent with chemical reactions. For example, in a gas-free system the thermal conductivity of the substance inside of the combustion wave depends upon the extent of substance conversion (extent of burnout), and its heat capacity depends on temperature. At the same time, the rate of substance conversion in this reaction is a function of both the temperature and extent of substance conversion [1].

The intensification of transfer processes often presumes their involved interaction with the surrounding medium. This, in turn, leads to the appearance of complicated initial conditions and time-varying boundary conditions in the mathematical formulation of the problem.

It is natural that, when these phenomena are described by linear models, this leads to a considerable discrepancy between the predicted results and the actual behaviour of the process due to the errors of linearization. On the other hand, the use of non-linear models involves computational difficulties because of the inadequacy of the mathematical apparatus applied in this field.

It was shown earlier that some of these difficulties could be overcome by employing variational methods. They have already manifested themselves as an effective and elegant tool in solving various problems of combined heat and mass transfer [2]. This paper is concerned with a further development of one of these methods—the method of generalized coordinates. As suggested, it allows one to considerably extend the range of combined heat and mass transfer problems accessible for solution, and to include non-linear non-stationary problems with complex boundary and initial conditions.

2. FORMULATION OF THE PROBLEM

The method will be considered on the example of a system of equations which describes the combined heat and mass transfer in an infinite plate for the case when the heat and mass conductivity coefficients and the temperature-gradient coefficient depend on temperature and on mass transfer potential. The assumption that the problem is symmetric and that the plane $X = 0$ passes through the middle of the plate yields

$$\frac{\partial \theta_1}{\partial Fo} = \frac{\partial}{\partial X}\left[(M_1 + Ko^* Pn\, Lu\, K)\frac{\partial \theta_1}{\partial X}\right] - Ko^* Lu \frac{\partial}{\partial X}\left(M_2 \frac{\partial \theta_2}{\partial X}\right)$$

$$\frac{\partial \theta_2}{\partial Fo} = -Lu\, Pn \frac{\partial}{\partial X}\left(K - \frac{\partial \theta_1}{\partial X}\right) + Lu \frac{\partial}{\partial X}\left(M_2 \frac{\partial \theta_2}{\partial X}\right) \quad (1)$$

where $M_1 = 1 + m_1\theta_1$, $M_2 = 1 + m_2\theta_2$, $K = 1 + k\theta_1$ ($m_1, m_2, k = \text{const.}$).

The functions $M_1(\theta_1)$ and $M_2(\theta_2)$ describe the

NOMENCLATURE

Bi_q	heat transfer Biot number
Fo	Fourier number (dimensionless time)
Ki_m	mass transfer Kirpichev number
Ki_q	heat transfer Kirpichev number
Ko	Kossovich number
Ko^*	$\varepsilon\, Ko$
Lu	Luikov number
Pn	Posnov number
X	space coordinate [dimensionless].

Greek symbols

ε	phase change criterion
$\theta_1(X, Fo)$	temperature [dimensionless]
$\theta_2(X, Fo)$	mass transfer potential [dimensionless].

Superscript

$(\cdot)$	$\partial/\partial Fo$.

dependence of the thermal conductivity coefficient of the medium on temperature and of the mass conductivity coefficient on mass transfer potential. The function $K(\theta_1)$ accounts for the linear temperature dependence of the product between the mass conductivity coefficient of the medium and the temperature-gradient coefficient.

To simplify the presentation of the method, symmetric problems of transfer will be considered, i.e. it will be assumed that

$$\frac{\partial \theta_i}{\partial X}(0, Fo) = 0 \quad (i = 1, 2). \tag{2}$$

The matching between internal transfer and the outer media is usually assumed in the form of boundary conditions of the first kind

$$\theta_i(1, Fo) = F_i(Fo) \quad (i = 1, 2) \tag{3}$$

of the second kind

$$-\frac{\partial \theta_1}{\partial X}(1, Fo) + Ki_q(Fo) = 0$$

$$-\frac{\partial \theta_2}{\partial X}(1, Fo) + Pn\frac{\partial \theta_1}{\partial X}(1, Fo) + Ki_m(Fo) = 0 \tag{4}$$

or of the third kind

$$\frac{\partial \theta_1}{\partial X}(1, Fo) - Bi_q[1 - \theta_1(1, Fo)] + (1-\varepsilon) Ko\, Lu\, Ki_m(Fo) = 0$$

$$-\frac{\partial \theta_2}{\partial X}(1, Fo) + Pn\frac{\partial \theta_1}{\partial X}(1, Fo) + Ki_m(Fo) = 0. \tag{5}$$

The initial conditions of the problem are

$$\theta_i(X, 0) = \Phi_i(Fo) \quad (i = 1, 2). \tag{6}$$

Here, Ki_q, Ki_m, F_i, Φ_i are considered to be the known functions of time.

3. VARIATIONAL ANALOGUE OF THE PROBLEM

Application of variational methods to the solution of boundary-value problems for differential equations is based on the analogy that exists between the differential and variational statements of the problem: it is necessary (if possible) to select such a functional for which equation (1), together with the corresponding boundary conditions, could constitute the Euler equations. However, it was found in ref. [2] that there was no such functional for the problems of type (1)–(6). At the same time it suggested a number of non-classic variational approaches to the solution of non-linear non-stationary problems of the same type. One of these, generalizing the Biot method in the heat conduction theory, formulates the initial problem in terms of the so-called generalized coordinates.

Using the additional form of the variational principle for the combined heat and mass transfer phenomena [2], it is possible to show that to equations (1) there corresponds a system of Lagrange equations stated for generalized coordinates:

$$\begin{aligned}
&\int_0^1 \left(\dot\theta_1 \frac{\partial \dot\theta_1}{\partial \dot q_j} + \dot\theta_2 \frac{\partial \dot\theta_2}{\partial \dot q_j}\right) dX \\
&+ \int_0^1 \left[(M_1 + Ko^*\, Pn\, Lu\, K)\frac{\partial \theta_1}{\partial X}\frac{\partial^2 \theta_1}{\partial X\, \partial q_j} - Ko^*\, Lu\, M_2 \frac{\partial \theta_2}{\partial X}\frac{\partial^2 \theta_1}{\partial X\, \partial q_j} - Lu\, Pn\, K \frac{\partial \theta_1}{\partial X}\frac{\partial^2 \theta_2}{\partial X\, \partial q_j} + Lu\, M_2 \frac{\partial \theta_2}{\partial X}\frac{\partial^2 \theta_2}{\partial X\, \partial q_j}\right] dX \\
&= \left[\left((M_1 + Ko^*\, Pn\, Lu\, K)\frac{\partial \theta_1}{\partial X} - Ko^*\, Lu\, M_2 \frac{\partial \theta_2}{\partial X}\right)\frac{\partial \theta_1}{\partial q_j} - Lu\left(Pn\, K \frac{\partial \theta_1}{\partial X} - M_2 \frac{\partial \theta_2}{\partial X}\right)\frac{\partial \theta_2}{\partial q_j}\right]_{X=1} \\
&- \left[\left((M_1 + Ko^*\, Pn\, Lu\, K)\frac{\partial \theta_1}{\partial X} - Ko^*\, Lu\, M_2 \frac{\partial \theta_2}{\partial X}\right)\frac{\partial \theta_1}{\partial q_j} - Lu\left(Pn\, K \frac{\partial \theta_1}{\partial X} - M_2 \frac{\partial \theta_2}{\partial X}\right)\frac{\partial \theta_2}{\partial \theta_j}\right]_{X=1} \\
&\qquad (j = \overline{1, m}).
\end{aligned} \tag{7}$$

Here $q_j = q_j(Fo)$ are the generalized coordinates. Note that by virtue of symmetry conditions (2), the expression within square brackets vanishes at $X = 0$, and this will be taken into account in what follows.

It was shown earlier that in the problems of combined heat and mass transfer the generalized coordinates may be such parameters of the system as the thickness of the thermal or mass layers, height of the wave front, the values of transfer potentials in the central part of the medium, etc. The incorporation of generalized coordinates with the functions that approximate the potentials of transfer allows the transition from equations (7) to a Cauchy problem for the system of ordinary differential equations, thus making it possible to obtain simple analytical solutions for a number of problems. One other approach to the use of generalized coordinates will now be considered.

4. VARIATIONAL-DIFFERENTIAL PROCEDURE

Introduce the $m = 2(n+1)$ generalized coordinates and consider that the first $n+1$ of them describe the thermal properties of the system (they will be denoted by $q_{11}, q_{12}, \ldots, q_{1,n+1}$) and the rest describe the dynamics of mass transfer in the system (these will be denoted by $q_{21}, q_{22}, \ldots, q_{2,n+1}$). The segment [0, 1] will be divided into n parts. In the general case, the splitting can be arbitrary, but in the present case it will be assumed for simplicity that the space region is divided into n parts of length h. The points of splitting will be designated as

$$X_i(X_i = ih \quad (i = 1,2,\ldots,n+1), \quad X_1 = 0, X_{n+1} = 1)$$

and the following notation will be adopted:

$$R_{ki} = \int_{X_i}^{X_{i+1}} \theta_k \frac{\partial \dot{\theta}_k}{\partial \dot{q}_{kj}} \mathrm{d}X \quad (k = 1,2; i,j = \overline{1,n}),$$

$$G_{1i} = \int_{X_i}^{X_{i+1}} \left[(M_1 + Ko^* Pn\, Lu\, K) \frac{\partial \theta_1}{\partial X} \frac{\partial^2 \theta_1}{\partial X \partial q_{1j}} - Ko^* Lu\, M_2 \frac{\partial \theta_2}{\partial X} \frac{\partial^2 \theta_1}{\partial X \partial q_{1j}} \right] \mathrm{d}X$$

$$G_{2i} = -\int_{X_i}^{X_{i+1}} \left[Lu\, Pn\, K \frac{\partial \theta_1}{\partial X} \frac{\partial^2 \theta_2}{\partial X \partial q_{2j}} - Lu\, M_2 \frac{\partial \theta_2}{\partial X} \frac{\partial^2 \theta_2}{\partial X \partial q_{2j}} \right] \mathrm{d}X \quad (i,j = \overline{1,n})$$

$$P_1 = \left[(M_1 + Ko^* Pn\, Lu\, K) \frac{\partial \theta_1}{\partial X} - Ko^* Lu\, M_2 \frac{\partial \theta_2}{\partial X} \right] \frac{\partial \theta_1}{\partial q_{1,n+1}} \Bigg|_{X=1}$$

$$P_2 = -Lu \left(Pn\, K \frac{\partial \theta_1}{\partial X} - M_2 \frac{\partial \theta_2}{\partial X} \right) \frac{\partial \theta_2}{\partial q_{2,n+1}} \Bigg|_{X=1} \tag{8}$$

Then the system of expressions (7) can be written as

$$\begin{cases} \sum_{i=0}^{n} R_{1i} + \sum_{i=0}^{n} G_{1i} = P_1 \\ \sum_{i=0}^{n} R_{2i} + \sum_{i=0}^{n} G_{2i} = P_2. \end{cases} \tag{9}$$

The generalized coordinates are taken to be the values of the potentials of transfer at the points of splitting of the space region:

$$q_{ki}(Fo) = \theta_k(X_i, Fo) \quad (k = 1,2; i = \overline{1,n+1}).$$

Denote the generalized temperature $\theta_1^{i,i+1}$, and the mass transfer potential over the segment $[X_i, X_{i-1}]$ by $\theta_2^{i,i+1}$.

The approximating functions will be introduced according to projection-grid methods [4] at each splitting segment separately, expressing explicitly the space component in terms of these functions and non-explicitly the time component in terms of the generalized coordinates fixed with respect to only his splitting segment. That is, it will be assumed that

$$\theta_k^{i,i+1}(X, Fo) = \phi(X, q_{ki}, q_{k,i+1}) \quad (k = 1,2; i = \overline{1,n}). \tag{10}$$

Then, in equations (9) the components containing derivatives with respect to the generalized coordinates, which are not fixed at this segment, will vanish and these equations can be stated in the form

$$\begin{cases} R_{k1} + G_{k1} = 0 \\ R_{k1} + R_{k2} + G_{k1} + G_{k2} = 0 \\ \cdots\cdots\cdots\cdots \\ R_{k,n-1} + R_{kn} + G_{k,n-1} + G_{kn} = 0 \\ R_{kn} + G_{kn} = P_k \quad (k = 1,2). \end{cases} \tag{11}$$

As regards equations (11) it is possible to note that with the approximation in the form of equation (10) they represent a system consisting of $2(n+1)$ first-order ordinary differential equations for $2(n+1)$ generalized coordinates q_{1i}, q_{2i}. Actually, integration over the space component in equations (8) converts the terms G_{ki} to the form of the function of q_{kj}, whereas the terms R_{ki} become the functions of both q_{kj} and $\dot{q}_{kj}$. Since the derivatives $\partial \dot{\theta}_i / \partial \dot{q}_{ij}$ do not depend on $\dot{q}_{ij}$, the terms R_{ki} involve only the first power of the time derivatives of generalized coordinates. Thus, this yields a system of first-order governing equations the initial conditions for which follow from expressions (6). Account of boundary conditions in the system of governing equations is taken by means of transition in equations (3), (4) or (5), on the basis of approximating functions (10), to the generalized coordinates and inclusion of the expressions, obtained for the coordinates q_{in} and $q_{i,n+1}$, into the terms P_i of the governing systems. Then, the solution of this system for q_{ij} and substitution of the resulting solutions into equation (10) will yield the final solution of the problem.

5. THE PROBLEMS OF COMBINED HEAT AND MASS TRANSFER

As an example, consider the simplest procedure in the approximation of transfer potentials—linear approximation on the generalized coordinates. Equation (10) can be written in the form

$$\theta_k^{i,i+1}(X,Fo)=\frac{X-X_i}{h}(q_{k,i+1}-q_{ki})+q_{ki}$$

$$(k=1,2;i=\overline{1,n}). \tag{12}$$

Substituting equation (12) into equation (8) and carrying out the required operations, it is possible to rewrite the system of equations (11) in the following form:

$$\frac{1}{3}\dot{q}_{11}+\frac{1}{6}\dot{q}_{12}=\frac{1}{h^2}G_{11}$$

$$\frac{1}{6}\dot{q}_{1,i-1}+\frac{2}{3}\dot{q}_{1i}+\frac{1}{6}\dot{q}_{1,i+1}=\frac{1}{h^2}G_{i1}\quad(i=\overline{2,n})$$

$$\frac{1}{6}\dot{q}_{1n}+\frac{1}{3}\dot{q}_{1,n+1}=-\frac{1}{h^2}(G_{1,n+1}-P_1)$$

$$\frac{1}{3}\dot{q}_{21}+\frac{1}{6}\dot{q}_{22}=-\frac{1}{h^2}G_{21}$$

$$\frac{1}{6}\dot{q}_{2,i-1}+\frac{2}{3}\dot{q}_{2i}+\frac{1}{6}\dot{q}_{2,i+1}=-\frac{1}{h^2}G_{2i}\quad(i=\overline{2,n})$$

$$\frac{1}{6}\dot{q}_{2n}+\frac{1}{3}\dot{q}_{2,n+1}=\frac{1}{h^2}(G_{2,n+1}-P_2). \tag{13}$$

In these equations the following notations are adopted:

$$G_{1i}=A_1Q_{1i}^{(1)}+\frac{T}{2}Q_{1i}^{(2)}-A_2\left(\frac{m_2}{2}Q_{2i}^{(2)}+Q_{2i}^{(1)}\right)$$

$$(i=\overline{1,n})$$

$$G_{1,n+1}=A_1Q_1^{(3)}+\frac{T}{2}Q_1^{(4)}-A_2\left(\frac{m_2}{2}Q_2^{(4)}+Q_2^{(3)}\right)$$

$$G_{2i}=A_3\left(\frac{k}{2}Q_{1i}^{(2)}+Q_{1i}^{(1)}-A_4\left(\frac{m_2}{2}Q_{2i}^{(2)}+Q_{2i}^{(1)}\right)\right.$$

$$(i=\overline{1,n})$$

$$G_{2,n+1}=A_3\left(\frac{k}{2}Q_1^{(4)}+Q_1^{(3)}\right)-A_4\left(\frac{m_2}{2}Q_2^{(4)}+Q_2^{(3)}\right)$$

$$Q_{ki}^{(1)}=q_{k,i+1}-q_{ki},\quad Q_{ki}^{(2)}=q_{k,i+1}^2-q_{ki}^2$$

$$(k=1,2;i=1)$$

$$Q_{ki}^{(1)}=q_{k,i-1}-2q_{ki}+q_{k,i+1}$$

$$Q_{ki}^{(2)}=q_{k,i-1}^2-2q_{ki}^2+q_{k,i+1}^2$$

$$(k=1,2;i=\overline{2,n})$$

$$Q_k^{(3)}=q_{k,n+1}-q_{kn},\quad Q_k^{(4)}=q_{k,n+1}^2-q_{kn}^2\quad(k=1,2)$$

$$P_1=(A_1+Tq_{1,n+1})Q_1^{(3)}-A_2(1+m_2q_{2,n+1})Q_2^{(3)}$$

$$P_2=A_3(1+kq_{1,n+1})Q_1^{(3)}-A_4(1+m_2q_{2,n+1})Q_2^{(3)}$$

$$A_1=1+Ko^*\,Pn\,Lu,\quad A_2=Ko^*\,Lu$$

$$A_3=Lu\,Pn,\quad A_4=Lu$$

$$T=m_1+Ko^*\,Pn\,Lu\,k.$$

The system of governing equations (13) has been constructed without taking account of the boundary conditions at $X=1$. For these to be included in equations (13), it is necessary to make transition to generalized coordinates in equations (3) and (4) or (5). Taking into account equation (12), it is possible to obtain for the boundary conditions of the first kind (3) that

$$q_{k,n+1}=F_k(Fo)\quad(k=1,2) \tag{14}$$

for the boundary conditions of the second kind (4) that

$$q_{k,n+1}=B_kh+q_{kn}\quad(k=1,2)$$

$$B_1=Ki_q(Fo);\quad B_2=Pn\,Ki_q(Fo)+Ki_m(Fo) \tag{15}$$

and for the boundary conditions of the third kind (5) that

$$q_{1,n+1}=(1+h\,Bi_q)^{-1}[q_{1n}+h(Bi_q-(1-\varepsilon)Ko\,Lu\,Ki_m)]$$

$$q_{2,n+1}=q_{2n}+Pn(q_{1,n+1}-q_{1n})+h\,Ki_m. \tag{16}$$

It follows from the last relations that for the boundary conditions of the first kind the approximating equations express the generalized coordinates on the boundary in terms of the functions which here specify the change of transfer potentials, whereas for the boundary conditions of the second and third kinds these approximating equations establish the relationship between the generalized coordinates on the boundary of the space region and in the zone directly adjacent to it.

The practical application of the method will be considered by taking two examples of the solution of a non-linear problem at the following similarity numbers: $Lu=2$, $Ko^*=0.33$, $Pn=0.5$. It will be assumed that $m_1=m_2=k=0.1$. Introduce, as an example, 42 generalized coordinates ($n=20$): the coordinates $q_{11},\ldots,q_{1,21}$ describe the thermal properties of the system and $q_{21},\ldots,q_{2,21}$ describe the mass transfer in the system. It is evident that in equation (13) $n=20$, and the general quantity of equations in the system is equal to 42. First, consider the problem under the boundary conditions of the first kind assuming in equation (3) that $F_i(Fo)=0$ and in equation (6) that $\Phi_i(X,0)=1$ ($i=1,2$). This problem will play the part of a test when estimating the accuracy, because its solutions by the other methods are known [2]. By virtue of the adopted boundary conditions and of equation (14), it is necessary to assume in equations (13) that $q_{1,21}=0$ and $q_{2,21}=0$. Their subsequent solution by the Runge–Kutta method with the initial conditions $q_{kj}(0)=1$ makes it possible to obtain the generalized coordinates some of which are presented

Table 1. The values of generalized coordinates obtained under the boundary conditions of the first kind

	$X = 0.0$	$X = 0.2$	$X = 0.4$	$X = 0.6$	$X = 0.8$
Fo	q_{11}	q_{15}	q_{19}	$q_{1,13}$	$q_{1,17}$
0.02	0.9981	0.9972	0.9968	0.9821	0.7218
0.04	0.9962	0.9928	0.9906	0.8887	0.5680
0.1	0.9906	0.9656	0.8764	0.6913	0.3905
0.2	0.9024	0.8621	0.7246	0.5379	0.3062
0.4	0.6335	0.5755	0.4836	0.3542	0.1879
0.6	0.4223	0.4098	0.3379	0.2488	0.1262
0.8	0.2722	0.2591	0.2209	0.1611	0.0850
1.0	0.1842	0.1753	0.1439	0.1087	0.0573
Fo	q_{21}	q_{25}	q_{29}	$q_{2,13}$	$q_{2,17}$
0.02	0.9976	0.9968	0.9917	0.9460	0.6762
0.04	0.9922	0.9870	0.9575	0.8409	0.5280
0.1	0.9319	0.9060	0.8169	0.6393	0.3587
0.2	0.8126	0.7753	0.6690	0.4946	0.2749
0.4	0.5326	0.5038	0.4285	0.3291	0.1703
0.6	0.3576	0.3381	0.2782	0.2001	0.1057
0.8	0.2265	0.2155	0.1837	0.1339	0.0706
1.0	0.1526	0.1452	0.1237	0.0900	0.0474

in Table 1. Figure 1 shows the solution obtained from equation (13) by the employed variational-differential method in comparison with the earlier solutions of the same problem by the modified methods of Ritz and Kantorovich. This comparison shows a good agreement between the results.

Now, consider the capabilities of the method when solving complex problems of transfer. Assume for the above-considered non-linear problem the variable boundary conditions of the third kind (5) in which

$$Ki_m(Fo) = Ki_m^0(1 + P\,Fo + R\,Fo^2) \qquad (17)$$

where Ki_m^0, P, R = const.

Together with the parabolic dependence of the Kirpichev number on time (17), the parabolic dependence on the space coordinates will also be assumed for the initial conditions (6):

$$\Phi_1(Fo) = V(1-X^2), \quad \Phi_2(Fo) = W(1-X^2) \qquad (18)$$

where V, W = const.

Additional specifications are the similarity numbers: $\varepsilon = 0.5$, $Ki_m^0 = 0.5$, $Bi_q = 0.2$ and the values of the constants: $P = 1.5$, $R = 1.14$, $V = 0.3$, $W = 0.2$. The substitution of equations (18) into (16), then of equations (16) into (13) and taking account of the initial conditions

$$q_{1j}(0) = V[1-((j-1)h)^2]$$

$$q_{2j}(0) = W[1-((j-1)h)^2]$$

$$(j = 1, 2, \ldots, 21)$$

will yield, at the given values of the similarity criteria, the solution which is partially presented in Table 2. Figure 2 shows the dependence of the dimensionless

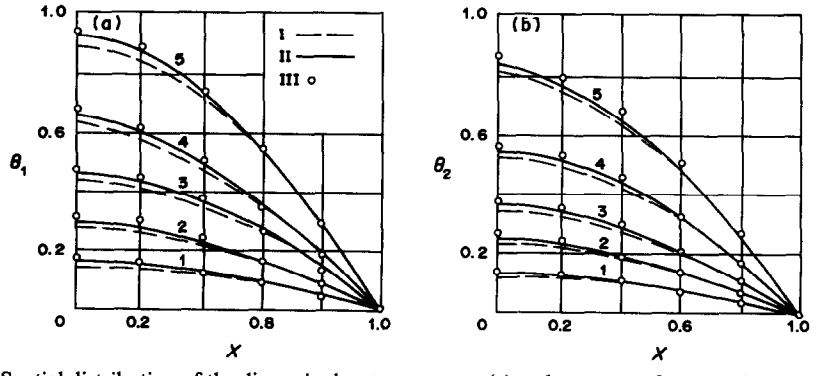

FIG. 1. Spatial distribution of the dimensionless temperature (a) and mass transfer potential (b) under the boundary conditions of the first kind at different values of Fo: 1, 1.0; 2, 0.8; 3, 0.6; 4, 0.4; 5, 0.2. I, solutions obtained by the method of generalized coordinates; II, by the modified Kantorovich method; III, by the modified Ritz method [2].

Table 2. The values of generalized coordinates obtained under the boundary conditions of the third kind and with the parabolic initial distribution of potentials

	$X = 0.0$	$X = 0.2$	$X = 0.4$	$X = 0.6$	$X = 0.8$
Fo	q_{11}	q_{15}	q_{19}	$q_{1,13}$	$q_{1,17}$
0.02	0.2893	0.2775	0.2425	0.1881	0.1417
0.04	0.2794	0.2684	0.2370	0.1956	0.1712
0.1	0.2600	0.2535	0.2376	0.2230	0.2238
0.2	0.2542	0.2535	0.2529	0.2568	0.2704
0.4	0.2808	0.2831	0.2905	0.3033	0.3222
0.6	0.3174	0.3201	0.3282	0.3417	0.3605
0.8	0.2757	0.2742	0.2716	0.0277	0.2839
1.0	0.1454	0.1478	0.1554	0.1692	0.1906
Fo	q_{21}	q_{25}	q_{29}	$q_{2,13}$	$q_{2,17}$
0.02	0.1960	0.1878	0.1621	0.1179	0.0703
0.04	0.1900	0.1807	0.1529	0.1121	0.0772
0.1	0.1619	0.1541	0.1315	0.1086	0.0908
0.2	0.1237	0.1204	0.1121	0.1024	0.0956
0.4	0.0808	0.0803	0.0792	0.0779	0.0771
0.6	0.0505	0.0504	0.0504	0.0504	0.0504
0.8	0.1814	0.1994	0.2550	0.3527	0.4973
1.0	0.6002	0.6220	0.6876	0.7969	0.9494

temperature and of the mass transfer potential on the value of Fo on the plate surface.

The analysis of the solutions reveals a complex behaviour of temperature fields and of the mass transfer potential under the adopted boundary conditions. At the initial instant the transfer potentials on the surface are equal to zero, whereas in the inner zone they differ from zero and increase from the surface to the middle of the plate according to the parabolic law. At small values of Fo, a sudden increase in θ_1 and θ_2 is observed both on the surface of the plate and in the region adjacent to it. Simultaneously, due to the internal gradients of the potentials, which still exceed their values in the surface region, there occurs a specific relaxation of the temperature and mass fields which leads to a decrease of θ_1 and θ_2 in the internal zone of the plate. For the potential θ_2 this occurs until $Fo = 0.62$, since here the initial sudden increase in the mass transfer potential on the surface is replaced, beginning from $Fo = 0.2$, by its slow decrease. However, starting from $Fo = 0.62$ almost a jumpwise increase of $\theta_2(1, Fo)$ is observed on the surface, and this leads to a rapid growth of the mass transfer potential in the inner zone when $Fo = 0.62$.

The dynamics of the temperature field in the given case are characterized by still greater complexity. The quantity $\theta_1(1, Fo)$ increases monotonously on the surface up to $Fo = 0.64$ and then begins to decrease monotonously. This leads to a slow decrease of temperature in the inner region of the plate. At $Fo = 0.12$ the temperature equilibrates over the entire spatial region and becomes equal to $\theta_1 = 0.25$. From this value of Fo up to $Fo = 0.7$ it grows in the inner zone and then decreases again from the value $\theta_1 = 0.33$ to 0.15 due to a decrease in the surface temperature.

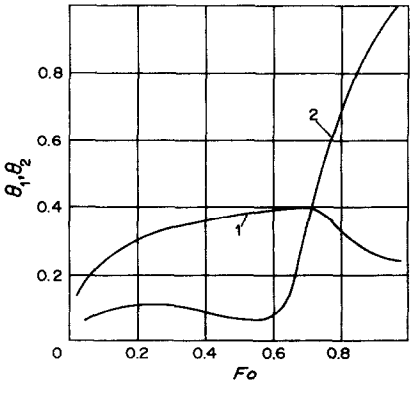

FIG. 2. Dependence of the dimensionless temperature (1) and mass transfer potential (2) on Fo on the surface of an infinite plate under the boundary conditions of the third kind and with the distribution of parabolic initial potentials.

6. CONCLUSION

The method of generalized coordinates allows one to solve the problems of combined non-linear non-stationary heat and mass transfer for time-dependent functions entering into boundary conditions and for complex, in form, initial distributions of transfer potentials. By selecting transfer potentials at fixed points in the medium as generalized coordinates, it is possible to obtain numerical–analytical solutions allowing theoretical analysis of the complex processes of combined heat and mass transfer.

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DEVELOPPEMENT DE LA METHODE DES COORDONNEES GENERALISEES POUR RESOUDRE LES PROBLEMES NON LINEAIRES DE TRANSFERT VARIABLE DE CHALEUR ET DE MASSE AVEC DES CONDITIONS AUX LIMITES COMPLEXES

Résumé—On développe la méthode des coordonnées généralisées pour l'appliquer à la résolution des problèmes de transfert combinés de chaleur et de masse, non linéaires et variables, avec des conditions aux limites complexes. On observe un bon accord entre les solutions avec des conditions aux limites de première espèce et celles obtenues par d'autres méthodes. On donne la solution du problème avec conditions du troisième ordre dépendantes du temps et avec une distribution initiale parabolique des potentiels.

DIE ENTWICKLUNG DER METHODE DER VERALLGEMEINERTEN KOORDINATEN ZUR LÖSUNG NICHTLINEARER INSTATIONÄRER PROBLEME DER WÄRME- UND STOFFÜBERTRAGUNG BEI KOMPLIZIERTEN RANDBEDINGUNGEN

Zusammenfassung—Die Methode der verallgemeinerten Koordinaten wird auf nichtlineare instationäre Wärme- und Stoffübertragungsprobleme mit komplizierten Randbedingungen angewandt. Die Übereinstimmung bei Randbedingungen erster Art mit den Ergebnissen nach anderen Verfahren ist gut. Die Lösung des Problems bei einer zeitabhängingen Randbedingung dritter Art und bei einem zu Anfang parabolischen Verlauf wird angegeben.

РАЗВИТИЕ МЕТОДА ОБОБЩЕННЫХ КООРДИНАТ ДЛЯ РЕШЕНИЯ НЕЛНЕЙНЫХ ЗАДАЧ ВЗАИМОСВЯЗАННОГО ТЕПЛО- И МАССПЕРЕНОСА СО СЛОЖНЫМИ КРАЕВЫМИ УСЛОВИЯМИ

Аннотация—Метод обобщенных координат развит применительно к решениям нестационарных задач взаимосвязанного тепло- и массопереноса при сложных краевых условиях. Показано хорошее совпадение решений при граничных условиях I рода с решениями, полученными другими методами. Дано решение задачи при зависящих от времени граничных условиях III рода и параболическом начальном распредении потенциалов.